Environment & Principal-Agent approach

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Abstract. In this article we offer a different market approach as an effective tool for environmental policy. We suggest a creation of markets out of reforestation using Principal- Agent methodology as an initial measure. We analyze two bonded diffusion processes proper for our modeling. Similar technique can be applied to other environmental issues.

Key words: Environmental Improvements, Principal-Agent, Stochastic Optimization.

1 Introduction

The objective of this paper is to look at some of the existing methods that aim to halt environmental degradation and to propose a new one, which may prove to be more effective. The praiseworthy solution recommended by United Nations 2005 Millennium Project [14] (Environment and human well-being: a practical strategy) falls in steps of Agenda 21 accorded in Rio in 1992. Likewise, the proposed " and practical steps that governments and international agencies can undertake to operationalize the concept of environmental sustainability " ...) leave many questions unanswered. One of the key difficulties is the "forest management for protection and sustainable production " above all in developing countries that harbor tropical rain forest. The other is mitigation of global climate change by 'stabilizing greenhouse gasses'. The Kyoto Protocol, the most popular agenda for reducing pollution by trading carbon quotas, has been raising doubts if it in fact cleans the environment or just shifts polluting gasses around. It seems that modern environmental policy suffers from several significant weakness, and is not capable of dealing with the world's most serious environmental problems.

None of the advocated solution promises any kind of milestone shift in the endless efforts to stop accelerated destruction of nature.

In this study we would like to introduce a singular market solution based on direct interplay of finance and environmental conservation. This approach will allow to deal with each environmental issue individually by using separate funds.

Although it is only one component of broad and perilous environmental crisis, we will focus on deforestation since it is transcendent for human and nature well being. Besides, technology made possible to assess the number of trees or biomass in given areas by using satellite monitoring and other tools. To simplify the task we will discuss one numerical value calling it *number of trees*, though a combination of the real number of trees perhaps of different types with biomass would be more adequate.

The initial step of our proposal will be of distributing *good* environmental certificates to the inhabitants of given community free of charge. *Good* certificate is meant to stimulate and encourage positive environmental action like reforestation, restoration and conservation of biodiversity.

During the following stage these certificates may be freely bought by all interested agents. The last phase would allow the possibility to purchase both *good* and *bad* certificates. *Bad* certificates could stimulate environmentally harmful actions . In the view that the last stage is still not well-structured, and stands up to a future (game like) analysis; it will not be included in this paper.

Our modeling has the hierarchical structure of three levels:

1)Global optimization problem.

2)Local optimization problem.

3)Individual agent problem.

The first level consists of a distribution of local and global environmental funds that are intended for reforestation into particular projects. The next one is about choosing appropiate methods of using the funds in the emission of optimal *good* environmental certificates. Optimal, in this case, means that agent's action would maximize reforestation and conservation outcomes. The local goal should be based on the temporal mean of an asset, and the major difficulties, in our modeling, stem from the following: the awards gained by agents have to be limited by the original fund since no financial institution would accept the risk of loosing money. We assume that the funds (local, national and global) would be managed by a financial institution that would not charge any transaction cost or expect any profit. If any resources remain they would be forwarded to another projects. In this approach the interplay between specified financial institution and the agent constitutes the Principal-Agent model. We will see several specific features in our modeling, and we deem closed solution highly desirable. It can be easily shown , at least in simple economics games *cf.* Finus & Rundshagen [4], that our approach would generate *directly* Nash equillibrium as socially optimal. [1]

Other proposals like permits to pollute or transferable development rights, *cf.* [3] are essentially market approaches that set limits on environmentaly harmful activities. Similar method called *cap and trade* is used in Sulfur Dioxide Program in the United States. As commented by Blackman and Harrington [1] with reference to developing countries "tradable permits are generally not practical."

It is important to stress that we do not pretend to valuate environ-

[1] Consider the simplest example from [9]. There are two countries with pollutant emission levels E_1 and E_2 and and country cost of pollution $\frac{c_i}{2}\left(E_1+E_2\right)^2$. The costs of abatement are $b_i\left(\frac{a}{b}-E_i\right)^2$ with $b_1=\frac{b}{2}, b_2=\frac{b}{4}$. In this case Nash optimal emission levels are higher than socially optimal.

In the Principal-Agent setting we propose the emission of certificates that pay $S_1-\frac{c_1}{2}\left(\alpha E_1+E_2\right)^2$ and $S_2-\frac{c_2}{2}\left(E_1+\beta E_2\right)^2$, for fixed S_1, S_2. Results easily that one can choose α, β such that Nash optimal levels match socially optimal with no additional costs. The minimal funds required are exactly $\frac{b}{2}\left(\frac{a}{b}-E_1^*\right)^2$ and $\frac{b}{4}\left(\frac{a}{b}-E_2^*\right)^2$, where "$*$" stands for agent's optimal actions.

ment by endowing forest with market value. What we propose instead is the direct market out of *environmental improvements,* always when high reliability measurment of actual state could be ensured: for example the number of wind turbines. The “conditional carrot” approach as “Principal Agent” methodology came to be know in economy might be the only way to deal with the most serious environmental crisis. Actually, this approach has been already under way in combating pollution, like for example opening of high-ocupancy-vehicle lines to hybrids even there is only one person in the vehicle. However, it poses different optimization problem because initial customer’s decision remains stable over time.

We can not guarantee creation of millions of good new jobs over ten years as it does a New Apollo project, [11]. However the Principal-Agent method is indeed aiming at creating new investment opportunities that stimulate economic development of the region, and achieve great things for humans and environment.

The solution we propose will simultaneously benefit local communities and the wildlife. It will offer the transparency in the transfer of funds for conservation purposes. It is possible that with continued funding, small scale projects (like this) will provide a format for elaboration of conservation schemes on global scale.

We will introduce two models. In the first model we are able to solve the problem completely in the case of linear certificates. In the second model, using dual technique, we represent the optimal agent's action (under additional assumption and in the case of logaritmic award) in terms of the corresponding Lagrange multiplier.

To find it, one has to solve the equation involving the expectation of complicated but in a sense elementary functionals of Brownian Motion. We call this solution semi-closed.

We provide "soft" theoretical justification of both. Further analysis of their real applicability and calibration depends on experience. In our setting, to start the project, we do not need precise models! [2]. " Good" certificates would contribute positively to the environment in any circumstances!

It is convenient, however, to explain how relevant optimization problems could be handled.

Analysis of different models and solution of some optimization problems can be found in our earlier studies, [6],[12],[13].

[2]This is the main reason that in this study we do not venture to analyze more complex economic models.

2. Viewing the horizon

Despite the multiple conservation proposals, many countries particularly in the tropical regions that naturally harbor the greatest biological diversity, have not been able to slow present trends of environmental degradation.

The professionals in charge of conservation have a tendency to speak in general moral terms about the need to protect forest and prevent diversity deterioration.

The conservation course of action often hangs in the thin air imposed by the environmental law, and often is transformed into the single concept of nature free of human presence, quite trendy within the context of modern environmental ethics. This perception however is frequently at odds with local realities. There is no doubt that the complex interactions of biology, economics, social and technological factors have to be approached and solved in an ethical way. However, effective environmental policy requires more than shifts of consciousness from human oriented into nature oriented.

The assumption that once the site is designated as the national protected area its biodiversity is preserved proved short-sighted. The shelter of its legal status has not resolved the problems of land tenures and speculation, or stopped the harmful agricultural activities. A combination of a legal loopholes, lack of political will and weak enforcement capabilities are allowing widespread illegal logging of many precious species of trees like Chile's alerces (also known as Patagonian cypresses). This problem exists in many tropical countries where high deforestation rates, environmental degradation, and biological impoverishments affect protected areas, national parks and Biosphere Reserves. It is unrealistic to expect that major wild land conservation programs could be undertaken without an appraisal of the likely benefits to people who live in poverty alongside protected areas. No strategy, however excellent its basis in scientific research, can succeed unless it takes account of the external forces exerted by nearby peoples, who, having no other choice, will degrade or destroy reserves no matter how cleverly they are managed internally.

Accordingly, the overwhelming majority of proposals to conciliate economic progress and quality of life with the necessities of biological conservation have financial incentives attached to them. However, disbursement of the funds public or private is often insufficient or sporadic, and usually derailed. The international fund-lending institutions tend to promote unrestrained development threatening directly biological, ecological and cultural diversity. The governmental subsidies (local and national) frequently bring more harm than benefit.

The aid is used by power groups without changing local ideas and uses of the environment, [5]. Many conservation proposals focused exclusively on the alternative activities like industrial reforestation or intensive, multi-crop land use that may appeal to the healthy self-interest of the local people by providing trees and harvests of value to them. However, they missed the real connection between the complex community problems, external market pressures and biodiversity loss. As Pompa and Kaus [8] rightly put forward " All the terracing, green mulching, selective harvesting, field rotation, crop diversity, and reforesting in the world cannot help if the external consumption of natural resources continues to outpace local sustainable practices and to offer economic incentives that out-compete long term conservation benefits."

Ecologically friendly activities such as collecting wild fruits, rubber, nuts (non-timber products), including pharmaceutically active substances are money-loosing propositions that push the some plant species to the brink of extinction.

On the other hand, several policies related research result in increasingly complex models that generate never-ending debate about their applicability. We do not refer to famous Schrödinger's phrase that "nature resists imitations through models", but to the fact that models entries can hardly be observed or estimated. The use of statistics is hampered by the lack of specified knowledge about the ways ecosystem works and its spatial and temporal changes. This, combined with scarce information about the social factors that contribute to the degradation of ecosystems and long time scale of consequences of our actions, make sound decision making based on Cost-Benefit analysis, particularly difficult.

3 General Problem

We shall concentrate on the stage 1 when certificates are handed out free of charge. In the present study, the analysis of the extention to the second stage when certificates have price is automatical.

Consider static local goal with the horizon T. The general problem is the following: given fund S, emit (Principal) certificate F that after the optimal agents action $u(F)$, generates:

$$\max_F E\{\int_0^T f(s) X_s^{u(F)} ds\},$$

$X(s)$ represents the number of trees at time s in a given area, and f is a time function.

More formally:

Let u be agent's action adopted to the Brownian filtration that modifies the dynamic of the corresponding process X_s^u.

$$u = u(F, s),$$

where $F(X_s^u, s \leq T)$ is agent's award at time T.

$0 \leq F \leq S$, where S is the corresponding fund.

The problem consist in the maximization of two functionals stemming from:

1. Agent's optimization problem.

$$\sup_u \quad J_1(F, u) = J_1(F, u(F))$$

   J_1 being agent's monetary award.

2. Principal optimization problem that emits certificates F.

$$\sup_F \quad J_2(F, u(F)) = J_2(F^*, u(F^*))$$

$$J_1(F, u) = E\left(F(X_s^u, s \leq T) - \text{cost}\right)$$

$$J_2(F, u) = E\left(\int_0^T f(s) X_s^{u(F)} ds\right).$$

In our first model we will consider $f(s) \equiv 1$ only. Decreasing f would, however, be more suitable.

The Principal is here the financial institution in charge of the management of the fund. The Principal issues certificates to agents.

We assume that $X(s)$ can be monitored with desirable confiability.

Awards are limmited by funds. There are two general possibilities to work with bounded awards:

1. Set bounds on unbounded models.

2. Consider bounded models.

It seems that the second choice is easier to analyze. In the first choice it is unclear what to do after hitting the corresponding barrier and, moreover, after the optimal agents action the hitting time usually doesn't have explicite probability law. Moreover in several specific applications like reforestation issues, models are bounded by respective capacities. Therefore bounded models are more appropriate. In biological systems the Pearl-Verhulst model defined by

$$dV(t) = V(t)(1 - V(t))dt + V(t)dW(t)$$

represents the growth with saturation. However we need models with absolute saturation (bounded). Our examples meet these requirements.

Unlike the most studies on this kind of optimization problems, we shall assume agent's linear utility and thus without lossing the general approach, one certificate sold. On the other hand certificates should be traded producing possibly the concentration of capital in the hands of powerfull individuals able to stop environmental degradation (deforestation in our example).“ …human societies in rural Africa , like those elsewhere, typically have a hierarchical structure dominated by a few powerful individuals, who may wish to advance their own personal interest” *cf.* [7]. Without proper awards these personal interests would not encourage environmentaly friendly actions.

As an advance of more general analysis we will study two models with some restrictions on awards and optimal agent's actions. In both models we shall work with more transparent continuous time diffusions as an approximation of "discrete time reality."

4 Model 1:

The number of trees in a given area is represented by the diffusion process $X(t)$ defined by:

$$dX(t) = f[X(t)(k - X(t))]dW(t) + (u(t) - \beta)(k - X(t))dt,$$

$X(0) = x, \qquad 0 < x < k$, k is the capacity, and f is any function ensuring existence and uniqueness of the solution, cf [10]; and $f(0) = 0$. For example $f(x) = |x|^{\alpha}$, $\alpha \geq \frac{1}{2}$. We will impose conditions such that optimal agent action $u^{*}(t) \geq \beta$. In this case $0 \leq X(t) \leq k$. We will see that solutions of corresponding optimization problems will be f free.

To simplify the notation we shall set $k = 1$. We use this parametrization, factorizing $u(s)$ and β, to ensure the absolute saturation. Large X means no big environmental problems and consequently, lower deforestation rate, while interpreting $u(s)$ as an *effort*, this produces smaller changes in the drift. Another pattern will be considered in model 2. Here we will consider linear awards, and leave the general solution for further studies. In this case we can't expect closed formulas.

It is convenient to write the award as

$$-A\int_0^T(1-X(s))ds-\gamma(1-X(T)+S,$$

where $A,\gamma,S\geq 0$, and the agent's cost is $\delta\int_0^T u^2(s)(1-X(s))ds$. This cost can be seen as a compromise between both: quadratic (of effort) and linear (of actual state). The cost is smaller for larger X (reforestation is cheaper). For simplicity we set $\delta=1$. We have to choose optimal A and γ such that after the optimal agent's action $E\left(\int_0^T X_s^{u^*(A,\gamma)}ds\right)$ would be maximized.

Theorem 1

For linear certificates and under the condition that optimal agent's effort u is bounded for any $T>0$, the optimal linear certificate is: take A as large as possible with the following restrictions:

$A\leq\beta^2$

$TA+\gamma\leq S$

$\gamma-2\beta\geq 0.$

Proof:

The Bellman approach leads to the following equation for the value function:

$$F_t(x,t)+\left(\frac{F_x^2(x,t)}{4}-\beta F_x(x,t)+A+\frac{1}{2}[f(x(1-x))]F_{xx}(x,t)\right)(1-x)=0.$$

We will find the solution of the form $F(x,t)=h(t)(1-x)$ and so it will be f free. We get the following equation for $h(t)$:

$$h'(t)+(\frac{1}{2}h(t)+\beta)^2-\beta^2+A=0.$$

Setting $h(t)=4H(t)-2\beta$ we obtain trivial Riccati equation

$$H'(t)+H^2(t)=\frac{\beta^2-A}{4}.$$

In our setting it is desirable to ensure the existence of bounded agent's optimal action independent of T. This fact translates easily into $\beta^2\geq A$. So we have to solve the problem with the following constrains:

1. $A\leq\beta^2$
2. $TA+\gamma\leq S$
3. $\gamma-2\beta\geq 0$

The second and third conditions result from the fact that the total award in any case must be positive and, respectively, to ensure that

$u^*(t) \geq \beta, \quad \text{for} \quad 0 \leq t \leq T.$

Writing $H(t) = \frac{\varphi'(t)}{\varphi(t)}, \qquad \varphi(0) = 1, \qquad$ we have $\frac{\varphi''(t)}{\varphi(t)} = \frac{\beta^2 - A}{4}$,

and $if \quad \beta^2 > A$

$\varphi(t) = Be^{\alpha t} + (1-B)e^{-\alpha t}, \qquad \text{with } \alpha = \frac{\sqrt{\beta^2 - A}}{2},$

$$B = \frac{(4\alpha + 2\beta - \gamma)e^{-\alpha T}}{(4\alpha - 2\beta + \gamma)e^{\alpha T} + (4\alpha + 2\beta - \gamma)e^{-\alpha T}}$$

if $\beta^2 = A$, then the solution for φ is linear.

Now optimal $u^*(t) = -\frac{h(t)}{2} \geq \beta$. Elementary calculations show that $E\left(\int_0^\alpha X(s)ds\right)$ will be maximized if $\int_0^t \varphi^2(s)ds$ would take the minimal possible value. If $A < \beta^2$, this means the minimization with respect to A y γ of

$$\frac{1}{2\alpha}\{B^2(e^{2\alpha T} - 1) + (1-B)^2(1 - e^{-2\alpha T})\} + 2B(1-B)T.$$

After some elementary computer work we obtain that the solution is: Take A as large as possible.

5 Model 2:

The second model is defined by

$$dX(t) = X(t)(k - X(t))dW(t) + (\delta u(t) - \beta)X(t)(k - X(t))dt$$

As before we set $k = 1, \delta = 1$, and for example $\beta = \frac{1}{2}, X(0) = \frac{1}{2}$. We are only able to obtain the semi-closed solution of agent's optimization problem in the case of an award $F(X(T))$ with special choise of F.

In this model the deforestation and effort effect produce larger changes of drift for middle sizes of $X(t)$.

Assume that the agent cost is $\int_0^T \frac{u^2(s)}{2} X(s)(1 - X(s)ds$. The motivation is similar to given in the previous model.

The solution will be based on heavy use of Girsanov's theorem that will change the law of corresponding process to the one of the auxiliary model:

$$Y(t) = \frac{1}{1 + e^{-W(t)}}.$$

By Itô's Lemma:

$$dY(t) = Y(t)(1 - Y(t))dW(t) + [\frac{1}{2}Y(t)(1 - Y(t)) - Y^2(t)(1 - Y(t))]dt.$$

Set $Z(t) = e^{\frac{1}{2}W(t) - \frac{1}{8}t}$.

Now

$$M(t) = X(t)Z(t) - \int_0^t u(s)Z(s)X(s)(1 - X(s))ds$$

is a martingale, $M(0) = x = \frac{1}{2}$.

Therefore $\forall z$, $E(-zM(T) + zx) = 0$.

Assume that the agent acts in locally optimal way. It means that at time s chooses the adopted action $u(s)$ that maximizes instantaneous drift minus cost if written as $H(u(s), W(s)) \cdot F(X(s))$; taking $X(s)$ as a constant.

The set of strategies that depends on $W(s)$ and $X(s)$ we shall call $\mathcal{U}$. Because $X(s)$ depends on $u(s)$, there is no reason that this procedure will produce globally optimal strategy.

Using standard dual method, our goal is to maximize with respect to $u(s) \in \mathcal{U}$

$$E(F(X(T)) - cost) =$$

$$E(F(X(T)) - \int_0^T \frac{u^2(s)}{2} X(s)(1 - X(s))ds) =$$

$$E((F(X(T)) - zX(T)Z(T) - \int_0^T \frac{u^2(s)}{2} X(s)(1 - X(s))ds$$
$$+z \int_0^T u(s)Z(s)X(s)(1 - X(s))ds + zx).$$

To make calculations more explicite we choose:

$$F(x) = \ln(x + B), B > 1.$$

Theorem 2 The optimal agent's locally optimal effort $u(t) = \check{z}Z(t)$ and optimal

$$X_T = \begin{cases} \frac{1}{\check{z}Z(t)} - B & \text{if } \frac{1}{\check{z}Z(t)} - B \in (0,1) \\ 0 & \text{if } \frac{1}{\check{z}Z(t)} - B \leq 0 \\ 1 & \text{if } \frac{1}{\check{z}Z(t)} - B \geq 1. \end{cases}$$

$\check{z}$ is the Lagrange multiplier and can be calculated from equation (1) that involves expectation(2) of functionals of Brownian motion given by (3) and (4).

Note

Using Girsanov's theorem, the maximization with no restriction on $u(s)$ at time s of the term $E \int_0^T \left[\left(zu(s)Z(s) - \frac{1}{2}u^2(s) \right) X(s)(1 - X(s)) \right] ds$

will require, the knowledge of $u(\tau), \tau \leq s$. We assume here that an agent has not such memory.

Proof: Standard approach *cf.* Cadenillas *et al* [2] gives formulas for $u(t)$ and $X(T)$.

The Lagrange multiplier $\check{z}$ is such that

$$E\left(Z(T)X(T) - \check{z}\int_0^T Z^2(s)(1-X(s))X(s)ds\right) = \frac{1}{2}. \qquad (1)$$

The main problem is to calculate

$$E\int_0^T Z^2 X(s)(1-X(s))ds = \tilde{E}\int_0^T e^{\frac{1}{4}s}X(s)(1-X(s))ds \qquad (2)$$

where under the "$\sim$" law

$$dX(t) = X(t)(1-X(t))dW(t) + [\check{z}e^{W(t)+\frac{1}{2}t} + \frac{1}{2}]X(t)(1-X(t))dt.$$

Firstly we take away the term $\check{z}e^{W(t)+\frac{1}{2}t}X(t)(1-X(t))$ using Girsanov's theorem.

$$\tilde{E}\left(\int_0^T e^{\frac{1}{4}s}X(s)(1-X(s))ds\right) =$$
$$E^*\left(\int_0^T e^{\frac{1}{4}s}X(s)(1-X(s))ds \cdot \mathcal{E}\left(\int_0^{\cdot} h(s)dW^*(s)\right)_T\right)$$

Where

$$h(s) = \frac{e^{W^*(s)+\frac{1}{2}s}}{\frac{1}{\check{z}} + \int_0^s e^{W^*(u)+\frac{1}{2}u}du}.$$

We use the usual notation:

For given martingale M, the notation $\mathcal{E}(M)_T$ means $e^{M_T - \frac{1}{2}<M,M>_T}$.

This change of measure needs justification. We leave it to the appendix.

Under P^* law

$$dX(t) = X(t)(1 - X(t))dW^*(t) + \frac{1}{2}X(t)(1 - X(t))dt$$

which is much closer to our auxiliary process.

Secondly and lastly we use once again Girsanov's theorem to express the latter expectation as

$$E\{\int_0^T e^{\frac{1}{4}s}X(s)(1-X(s))ds \cdot \mathcal{E}\left(\int_0^{\cdot} \tilde{h}(s)d(W(s) - \int_0^s X(u)du)\right)_T \cdot \mathcal{E}\left(\int_0^{\cdot} X(s)dW(s)\right)_T\},$$

where $\tilde{h}(s)$ is $h(s)$ but with $W^*(s)$ replaced by $W(s) - \int_0^s X(u)du$.

Under this new law

$$X(s) = \frac{1}{e^{-W(s)} + 1}.$$

Making the corresponding replacement for $W^*(s)$, we need to calculate:

$$E\{(\int_0^T e^{W(s)+\frac{1}{4}s}(e^{W(s)} + 1)^{-2} ds) \tag{3}$$

$$\times \exp[-\int_0^T h(s) + (e^{W(s)} + 1)^{-1} dW(s)]$$

$$\times \exp[-\int_0^T h(s)(1 + \frac{1}{2}h(s)) + (2e^{W(s)} + 2)^{-1} ds]\},$$

where

$$h(s) = \frac{\check{z} \exp[-W(s) + \frac{1}{2}s - \int_0^s (e^{W(u)} + 1)^{-1} du]}{1 + \check{z} \int_0^s \exp[-W(u) + \frac{1}{2}u - \int_0^u (e^{W(\tau)} + 1)^{-1} d\tau] du}. \tag{4}$$

This formula represents what we called previously semi-closed solution.

Using Bellman principle it is easy to obtain Parcial Differential Equation for agent's optimal cost function $W(x, t)$ in many similar models. In Model 2 it takes form:

$$0 = 2W_t'(x,t) + \left[W_{xx}''(x,t)x(1-x) - (W_x'(x,t))^2 - W_x'(x,t)\right]x(1-x)$$

$$W(x,T) = -ln(x+B).$$

6 Appendix

Without loss of generality we may assume $\check{z} = 1$ To justify the second change of measure we need to prove that

$$H_T = \exp[\int_0^T V(s)dW(s) - \frac{1}{2}\int_0^T V^2(s)ds]$$

is true martingale and not only local martingale.

$$V(s) = \frac{e^{W(s)} + \frac{1}{2}s}{1 + \int_0^s e^{W(u)+\frac{1}{2}u}du}$$

satisfies

$$dV(t) = V(t)(1 - V(t)dt + V(t)dW(t).$$

The law of this process we will call P. The best way to prove that $H(t)$ is true martingale is through some equivalences of laws of processes on $\mathcal{F}_T^V, \quad \forall T$.

Note that $V(t)$ is Pearl-Verhulst model.

Let under the law Q, $dV(t) = V(t)dt + V(t)dW(t)$. The law Q is equivalent to the law Q_1. Under Q_1, $V(t) = e^{W(t)-\frac{1}{2}t}$.

Now, under Q_1

$$Z_T = \exp[-\int_0^T V(s)dW(s) - \frac{1}{2}\int_0^T V^2(s)ds]$$

$$= \exp[-e^{W(T)-\frac{1}{2}T} + 1 - \frac{1}{2}\int_0^T e^{2W(s)-s}ds].$$

Z_T is clearly true martingale and by Girsanov theorem changes the law Q_1 into $\tilde{Q}$ such that under $\tilde{Q}$,

$$dV(t) = -V^2(t)dt + V(t)dW(t).$$

Moreover $\tilde{Z}_T > 0$, under $\tilde{Q}$ measure.

Therefore $Q \sim Q_1 \sim \tilde{Q} \sim P$ on $\mathcal{F}_T$, and we have proved that Z_T is P martingale. The last equivalence is obvious.

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